3/29/17

Test 3 on Thursday (tomorrow)
Quarter Test Fri 4/7

Page 414

17a) $\int_0^3 x \, dx$

"Find the antiderivative of x and then plug in 3, plug in 0 and subtract."

$= \frac{x^2}{2}\Big]_0^3$

$= \left(\frac{3^2}{2}\right) - \left(\frac{0^2}{2}\right)$

$= \frac{9}{2} - 0 = \frac{9}{2}$

18a) $\int_0^2 \left(1 - \frac{1}{2}x\right) dx$

$1x^0$

$\frac{1x^1}{1} = x$

$= x - \frac{1}{2} \cdot \frac{x^2}{2}\Big]_0^2$

$= x - \frac{x^2}{4}\Big]_0^2$

$= \left(2 - \frac{2^2}{4}\right) - \left(0 - \frac{0^2}{4}\right)$

$= 1 - 0 = 1$

19a) $\int_0^5 2x^0 \, dx$

$2 \qquad 2x + c$

$= 2x\Big]_0^5$

$= (2(5)) - (2(0))$

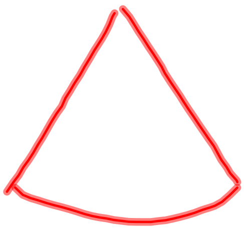
$= 10 - 0$

$= 10$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Review Sheet:

1)



$r = 2h$

$d = 40$

$r = 20$

$h = 10$

$\frac{dV}{dt} = 5$

Find: $\frac{dh}{dt}$

Cone

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi (2h)^2 h$

$V = \frac{1}{3}\pi 4h^2 h$

$V = \frac{1}{3}\pi 4h^3$

Derivative

$\frac{dV}{dt} = \frac{4}{3}\pi 3h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = 4\pi h^2 \frac{dh}{dt}$

$5 = 4\pi(10)^2 \frac{dh}{dt}$

$5 = 400\pi \frac{dh}{dt}$

$\frac{5}{400\pi} = \frac{dh}{dt}$

$\boxed{\frac{1}{80\pi} \text{ m/sec} = \frac{dh}{dt}}$

2) $\int \left(-2x^{-3}+20x^{-5}\right)dx$

Find the Anti-derivative

$$= \cancel{-2}^{\,}\frac{x^{-2}}{\cancel{-2}} + \cancel{20}^{-5}\frac{x^{-4}}{\cancel{-4}} + c$$

$$= x^{-2} - 5x^{-4} + c$$

$$= \frac{1}{x^2} - \frac{5}{x^4} + c$$

3) $\int \left(\frac{\cancel{-14}^{-7}x^{5/2}}{\cancel{2}}\right)dx = \int \left(-7x^{5/2}\right)dx$

$$= -7\frac{x^{7/2}}{7/2} + c = \frac{2}{\cancel{7}}\left(\cancel{-7}^{-1}x^{7/2}\right)+c$$

$$= -2x^{7/2} + c$$

$$= \boxed{-2\sqrt{x^7} + c}$$

4) $\int \left(\frac{-5 \sqrt[3]{x^2}}{3} \right) dx = \int \left(\frac{-5}{3} x^{2/3} \right) dx$

$= \cancel{\frac{-5}{3}}^{-1} \frac{x^{5/3}}{\cancel{\frac{5}{3}}} + c$

$= -x^{5/3} + c$

5) $\int_{-1}^{3} (-x^3 + 3x^2 + 1)\, dx$

$= -\frac{x^4}{4} + \frac{3x^3}{3} + 1x + c \Big]_{-1}^{3}$

$= -\frac{1}{4}x^4 + x^3 + x \cancel{+c} \Big]_{-1}^{3}$

$= \left(-\frac{1}{4}(3)^4 + (3)^3 + 3 \cancel{+c}\right) - \left(-\frac{1}{4}(-1)^4 + (-1)^3 + -1 \cancel{+c}\right)$

$= \left(\frac{-81}{4} + 30\right) - \left(-\frac{1}{4} - 2\right)$

$= \frac{39}{4} - \frac{-9}{4}$

$= \frac{48}{4} = 12$

6) $\int_{-3}^{0} (4\sqrt[3]{x})\,dx = \int_{-3}^{0} (4x^{1/3})\,dx$

$= 4\frac{x^{4/3}}{4/3} + c \Big]_{-3}^{0}$

$= 3x^{4/3}\Big]_{-3}^{0}$

$= \left(3(0)^{4/3}\right) - \left(3(-3)^{4/3}\right)$ $\quad (-3)^{4/3}$

$\sqrt[3]{81}$

$3\sqrt[3]{3}$

$= 0 - \left(3\left(3\sqrt[3]{3}\right)\right)$

$= -9\sqrt[3]{3} \approx 12.980$

7) $\frac{dy}{dx} = \frac{6x^2 - 2x^3}{x}$ $\quad y(1) = 4$

$f'(x) = \frac{6x^2 - 2x^3}{x}$ $\quad f(1) = 4$

$f'(x) = \frac{6x^2}{x} - \frac{2x^3}{x}$

$f'(x) = 6x - 2x^2$

$f(x) = \frac{6x^2}{2} - \frac{2x^3}{3} + c$

$f(x) = 3x^2 - \frac{2}{3}x^3 + c$

use $f(1)=4$

$4 = 3(1)^2 - \frac{2}{3}(1)^3 + c$

$4 = 3 - \frac{2}{3} + c$

$4 = \frac{7}{3} + c$

$-\frac{7}{3} \quad -\frac{7}{3}$

$\frac{5}{3} = c$

$f(x) = 3x^2 - \frac{2}{3}x^3 + \frac{5}{3}$

$\boxed{y = 3x^2 - \frac{2}{3}x^3 + \frac{5}{3}}$

8) $f'(x) = 3x^2 - 8x + 1$ $f(1) = 4$

$$f(x) = \frac{3x^3}{3} - \frac{8x^2}{2} + x + c$$

$$f(x) = x^3 - 4x^2 + x + c$$

$$4 = 1^3 - 4(1)^2 + 1 + c$$

$$4 = 1 - 4 + 1 + c$$

$$6 = c$$

$$f(x) = x^3 - 4x^2 + x + 6$$

9) $f''(x) = 6x^2 - 12x + 2$

$f'(x) = \frac{6x^3}{3} - \frac{12x^2}{2} + 2x + c$

$f'(x) = 2x^3 - 6x^2 + 2x + c$

use $f'(1) = -3$

$-3 = 2(1)^3 - 6(1)^2 + 2(1) + c$

$-3 = 2 - 6 + 2 + c$

$-1 = c$

$f'(x) = 2x^3 - 6x^2 + 2x - 1$

$f(x) = \frac{2x^4}{4} - \frac{6x^3}{3} + \frac{2x^2}{2} - 1x + c$

$f(x) = \frac{1}{2}x^4 - 2x^3 + x^2 - x + c$

use $f(-2) = 1$

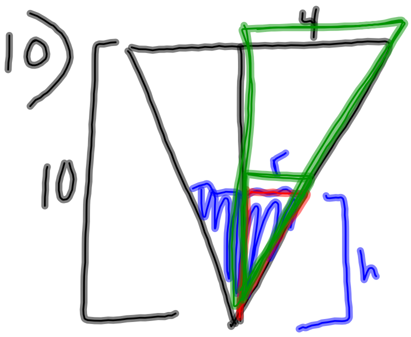
$1 = \frac{1}{2}(-2)^4 - 2(-2)^3 + (-2)^2 - (-2) + c$

$1 = 8 + 16 + 4 + 2 + c$

$-29 = c$

$f(x) = \frac{1}{2}x^4 - 2x^3 + x^2 - x - 29$

10)



Given $\frac{dV}{dt} = 2$ $h = 5$

Find: $\frac{dh}{dt}$

$\frac{4}{10} = \frac{r}{h}$

$\frac{2h}{5} = r$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \left(\frac{2h}{5}\right)^2 \cdot h$

$V = \frac{1}{3}\pi \frac{4h^2}{25} \cdot h$

$V = \frac{\pi}{3} \cdot \frac{4h^3}{25}$

$V = \frac{4\pi}{75} \cdot h^3 \rightarrow \frac{dV}{dt} = \frac{4\pi}{75} \cdot 3h^2 \frac{dh}{dt}$

$2 = \frac{4\pi}{75} 3(5)^2 \frac{dh}{dt}$

$2 = \frac{4\pi}{\cancel{75}} \cdot \cancel{75} \frac{dh}{dt}$

$2 = 4\pi \cdot \frac{dh}{dt}$

$\frac{2}{4\pi} = \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{1}{2\pi}$ ft/min